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CONSTRUCTION OF DISCONTINUOUS INTERLINEATION POLYNOMIAL SPLINES FOR FUNCTIONS OF TWO VARIABLES

The article suggests a general method for constructing discontinuous interlineation polynomial splines, which, as a partial case, include discontinuous and continuously differentiable splines. The theorems on interlineation and approximation properties of such discontinuous structures are formulated and proved. On the basis of the constructed discontinuous splines, a method for restoration of functions of two variables with first kind discontinuities is created. The theorems on the error of the approximation of discontinuous functions by the constructed discontinuous interlineation splines are proved. Examples are given.

Key words: discontinuous function, spline, interlineation, first kind discontinuity, interpolation.

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ПОБУДОВА РОЗРИВНИХ ІНТЕРЛІНАЦІЙНИХ ПОЛІНОМІАЛЬНИХ СПЛАЙНІВ ДЛЯ ФУНКЦІЙ ДВОХ ЗМІННИХ

Запропоновано загальний метод побудови розривних інтерлінаційних поліноміальних сплайнів, які як частинний випадок включають в себе розривні та неперервно-диференційовні сплайни. Сформульовані та доведені теореми про інтерлінаційні та апроксимаційні властивості таких розривних конструкцій. На основі побудованих розривних сплайнів створений метод відновлення функцій двох змінних, що мають розриви першого роду. Доведені теореми про похибку наближення розривних функцій побудованими розривними інтерлінаційними сплайнами. Наведені приклади.

Ключові слова: розривна функція, сплайн, інтерлінація, розрив першого роду, інтерполяція.

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ПОСТРОЕНИЕ РАЗРЫВНЫХ ИНТЕРЛИНАЦИОННЫХ ПОЛИНОМИАЛЬНЫХ СПЛАЙНОВ ДЛЯ ФУНКЦИЙ ДВУХ ПЕРЕМЕННЫХ

Предложен общий метод построения разрывных интерлинационных полиномиальных сплайнов, которые как частный случай включают в себя разрывные и непрерывно-дифференцируемые сплайны. Сформулированы и доказаны теоремы об интерлинационных и аппроксимационных свойствах таких разрывных конструкций. На основе построенных разрывных сплайнов создан метод восстановления функций двух переменных, имеющих разрывы первого рода. Доказаны теоремы о погрешности приближения разрывных функций построенными разрывными интерлинационными сплайнами. Приведены примеры.

Ключевые слова: разрывная функция, сплайн, интерлинация, разрыв первого рода, интерполяция.

Introduction. The problems of two–, three–, and four–dimensional computer tomography were studied in [1, 2] in detail. Computational experiments were carried out on the example of the human brain and moving human heart. The tomograms upcoming from the real functioning computer tomograph were used as input data. But the shortcoming of the method developed was the assumption that the objects studied were continuous. In fact the contemporary methods of computer tomography do not use the information about the internal structure of the human body, which is not the case in reality since each internal organ (such as stomach, liver, pancreas, ridge, etc.) has its own shape and tissue density. Hence, we deal with discontinuous objects.

Consider the following example. One of the more complicated problems of solid mechanics is studying the cracks at the internal points of a solid, where the *cracks* are actually the inclusions at the internal points of a body void of the material composing the body proper. One can say that the density of such a body is discontinuous, i.e. the density outside the cracks differs from that in the domain bounded by the crack walls.

Consequently, developing and studying the theory of approximation of discontinuous functions is relevant.

Analysis of recent studies. Nowadays, the main attention in the theory of functions of several variables is on the approximation of continuous and differentiable functions by continuous and differentiable splines. Nevertheless, practice shows that the multidimensional objects to be studied are mostly described by discontinuous functions. In [3] the precision of description of the internal structure of a 3D body is improved by taking into account the a-priory information about the body parts using appropriate functions of tree variables. Namely, the method interprets the information on the internal body structure as discontinuous function of three variables having discontinuities at the points of the surfaces bounding neighboring subdomains.

The whole development of computational and applied mathematics testifies that using any extra information about the object studied may lead to its more precise and quality reconstruction. For example, in [3] using the equation of the human skull surface is proposed for the precision reconstruction of the internal structure of a body.

Petukhov A. P. in his work [4] studies approximations of discontinuous functions in *the Hausdorff metric*. There exist methods for finding discontinuous solutions of boundary value problems developed in particular by *Serhiyenko I. V.*, *Deyneka V. S.*, *Skopets'kyy V. V.*, *Lytvyn O. M.* et al. [5]. In the paper by A. L. Aheev, T. V. Antonova [6] a method for determining the number of discontinuity points and their locations using *the Gibbs phenomenon* is proposed. Nevertheless, the method requires additional information, namely, the minimal and maximal jumps of the approximating function. Moreover, it is assumed that the intervals in which the Gibbs phenomena are located do not intersect, i.e. separating the points closely located is impossible.

The first step towards solving this problem was taken in [7 – 9], where the method for approximating discontinuous functions of two variables by discontinuous interpolation splines on rectangular and triangular node grid was proposed. In [10] an algorithm for determining the discontinuity lines for a function of two variables using discontinuous interpolants constructed was developed.

This paper deals with constructing and studying the discontinuous interlineation operators for approximating discontinuous functions of two variables by its known traces on a system of lines using rectangular elements in the case of known discontinuity lines. The method presented will allow finding the discontinuity lines of a function of two variables, determining the optimal nodes of the approximating discontinuous interlineation spline, and restoring inhomogeneous internal structure.

Problem setting. Let $f(x,y)$ be a discontinuous function in the domain D. Assume that the domain D is parted by straight lines $x_0 = 0 < x_1 < x_2 < ... < x_m = 1$, $y_0 = 0 < y_1 < y_2 < ... < y_n = 1$ into rectangular elements $\Pi_{ij} = (x_{i-1}, x_i) \times (y_{j-1}, y_j), i = \overline{1,m}, \; j = \overline{1,n}$. The function $f(x,y)$ and its derivatives up to the order $\rho - 1$ have the first kind discontinuities on the boundaries of these rectangular elements (possibly not all of them). It is required to construct a discontinuous spline such that interlineation and approximation properties hold.

Construction of discontinuous interlineation operator. We introduce the notations: $\varphi 1_i^+(y) = \lim\limits_{x \to x_i + 0} f(x,y)$, $\varphi 1_i^-(y) = \lim\limits_{x \to x_i - 0} f(x,y)$ for the trace of the function $f(x,y)$ on the lines $x = x_i$, $i = \overline{1,m}$. If $\varphi 1_i^+(y) = \varphi 1_i^-(y)$, then the function $f(x,y)$ is continuous on the line $x = x_i$, otherwise it has a discontinuity on the given line.

Consider the element $\Pi_{ij} = (x_{i-1}, x_i) \times (y_{j-1}, y_j), \; i = \overline{1,m}, \; j = \overline{1,n}$.

Definition 1. *The discontinuous interlineation polynomial spline* in the domain D corresponding to the given partition of D into subdomains Π_{ij} is the function:

$$S(x,y) = S_{ij}(x,y), \quad S_{ij}(x,y) = S1_{ij}(x,y) + S2_{ij}(x,y) - S12_{ij}(x,y), \quad (x,y) \in \Pi_{ij} \subset D, \tag{1}$$

where

$$S1_{ij}(x,y) = S1_{ij}(x,y; \{\varphi 1_{i-1,s}(y)\}; \{\varphi 1_{i,s}(y)\}, s = \overline{0,\rho-1}) = \sum_{s=0}^{\rho-1} \varphi 1^+{}_{i-1,s}(y) \cdot h1_{i-1,s}(x) + \sum_{s=0}^{\rho-1} \varphi 1^-{}_{i,s}(y) \cdot h1_{i,s}(x);$$

$$S2_{ij}(x,y) = S2_{ij}(x,y; \{\varphi 2_{j-1,p}(x)\}; \{\varphi 2_{j,p}(x)\}, p = \overline{0,\rho-1}) = \sum_{p=0}^{\rho-1} \varphi 2^+{}_{j-1,p}(x) \cdot h2_{j-1,p}(y) + \sum_{p=0}^{\rho-1} \varphi 2^-{}_{j,p}(x) \cdot h2_{j,p}(y);$$

$$S12_{ij}(x,y) = S12_{ij}(x,y; \{\varphi 1_{i-1,s}(y)\}; \{\varphi 1_{i,s}(y)\}, \; s = \overline{0,\rho-1}, \{\varphi 2_{j-1,p}(x)\}; \{\varphi 2_{j,p}(x)\}, \; p = \overline{0,\rho-1}) =$$
$$= \sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1} C^{++}_{i-1,j-1,s,p} h1_{i-1,s}(x) h2_{j-1,p}(y) + \sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1} C^{+-}_{i-1,j,s,p} h1_{i-1,s}(x) h2_{j,p}(y) +$$
$$+ \sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1} C^{-+}_{i,j-1,s,p} h1_{i,s}(x) h2_{j-1,p}(y) + \sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1} C^{--}_{i,j,s,p} h1_{i,s}(x) h2_{j,p}(y),$$

$h1_{k,s}(x), h2_{l,p}(y)$ are *the Hermit basic polynomials* of degree $2\rho - 1$ with the properties:

$$h1^{(s')}{}_{k,s}(x_{k'}) = \delta_{k,k'}\delta_{s,s'}, \; k,k' \in \{i-1,i\}, s,s' \in \{0,\rho-1\}, \quad h2^{(p')}{}_{l,p}(y_{l'}) = \delta_{l,l'}\delta_{p,p'}, l,l' \in \{j-1,j\}, p,p' \in \{0,\rho-1\}.$$

Theorem 1. *Assume*

$$\left(\varphi 1^+{}_{i,s}(y_j)\right)^{(p)} = \left(\varphi 2^+{}_{j,p}(x_i)\right)^{(s)} = C^{++}{}_{ijsp}, \; \left(\varphi 1^-{}_{i,s}(y_j)\right)^{(p)} = \left(\varphi 2^+{}_{j,p}(x_i)\right)^{(s)} = C^{-+}{}_{ijsp},$$
$$\left(\varphi 1^-{}_{i,s}(y_j)\right)^{(p)} = \left(\varphi 2^-{}_{j,p}(x_i)\right)^{(s)} = C^{--}{}_{ijsp}, \; \left(\varphi 1^+{}_{i,s}(y_j)\right)^{(p)} = \left(\varphi 2^-{}_{j,p}(x_i)\right)^{(s)} = C^{+-}{}_{ijsp}.$$

Then the function $S_{ij}(x,y)$ *satisfies the following properties on the boundary of the rectangle* Π_{ij} *:*

$$\left.\frac{\partial^{s'} S_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_{i-1}} = \varphi 1^+_{i-1,s'}(y), \quad \left.\frac{\partial^{s'} S_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_i} = \varphi 1^-_{i,s'}(y), \; y_{j-1} \le y \le y_j, \; s' = \overline{0,\rho-1}; \tag{2}$$

$$\left.\frac{\partial^{p'} S_{ij}(x,y)}{\partial y^{p'}}\right|_{y=y_{j-1}} = \varphi 2^+_{j-1,p'}(x), \quad \left.\frac{\partial^{p'} S_{ij}(x,y)}{\partial y^{p'}}\right|_{y=y_j} = \varphi 2^-_{j,p'}(x), x_{i-1} \le x \le x_i, \; p' = \overline{0,\rho-1}. \tag{3}$$

Proof. Substituting $x = x_{i-1}$ in (1) we get:

$$S_{ij}(x_{i-1},y)=S1_{ij}(x_{i-1},y)+S2_{ij}(x_{i-1},y)-S12_{ij}(x_{i-1},y),=\sum_{s=0}^{\rho-1}\varphi1^{+}{}_{i-1,s}(y)\cdot h1_{i-1,s}(x_{i-1})+\sum_{s=0}^{\rho-1}\varphi1^{-}{}_{i,s}(y)\cdot h1_{i,s}(x_{i-1})+$$

$$+\sum_{p=0}^{\rho-1}\varphi2^{+}{}_{j-1,p}(x_{i-1})\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\varphi2^{-}{}_{j,p}(x_{i-1})\cdot h2_{j,p}(y)-$$

$$-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,s,p}h1_{i-1,s}(x_{i-1})h2_{j-1,p}(y)+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{+-}_{i-1,j,s,p}h1_{i-1,s}(x_{i-1})h2_{j,p}(y)+$$

$$+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{-+}_{i,j-1,s,p}h1_{i,s}(x_{i-1})h2_{j-1,p}(y)+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{--}_{i,j,s,p}h1_{i,s}(x_{i-1})h2_{j,p}(y)=$$

$$=\sum_{s=0}^{\rho-1}\varphi1^{+}{}_{i-1,s}(y)\cdot\delta_{i-1,i-1}\delta_{s,0}+\sum_{s=0}^{\rho-1}\varphi1^{-}{}_{i,s}(y)\cdot\delta_{i,i-1}\delta_{s,0}+\sum_{p=0}^{\rho-1}\varphi2^{+}{}_{j-1,p}(x_{i-1})\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\varphi2^{-}{}_{j,p}(x_{i-1})\cdot h2_{j,p}(y)-$$

$$+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{-+}_{i,j-1,s,p}\delta_{i,i-1}\delta_{s,0}h2_{j-1,p}(y)+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{--}_{i,j,s,p}\delta_{i,i-1}\delta_{s,0}h2_{j,p}(y)=$$

$$=\varphi1^{+}{}_{i-1,0}(y)+\sum_{p=0}^{\rho-1}\varphi2^{+}{}_{j-1,p}(x_{i-1})\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\varphi2^{-}{}_{j,p}(x_{i-1})\cdot h2_{j,p}(y)-$$

$$-\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,0,p}h2_{j-1,p}(y)-\sum_{p=0}^{\rho-1}C^{+-}_{i-1,j,0,p}h2_{j,p}(y)=\left|\begin{matrix}C^{++}_{i-1,j-1,0,p}=\varphi2^{+}{}_{j-1,p}(x_{i-1})\\ C^{+-}_{i-1,j,0,p}=\varphi2^{-}{}_{j,p}(x_{i-1})\end{matrix}\right|=$$

$$=\varphi1^{+}{}_{i-1,0}(y)+\sum_{p=0}^{\rho-1}\varphi2^{+}{}_{j-1,p}(x_{i-1})\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\varphi2^{-}{}_{j,p}(x_{i-1})\cdot h2_{j,p}(y)-$$

$$-\sum_{p=0}^{\rho-1}\varphi2^{+}{}_{j-1,p}(x_{i-1})h2_{j-1,p}(y)-\sum_{p=0}^{\rho-1}\varphi2^{-}{}_{j,p}(x_{i-1})h2_{j,p}(y)=\varphi1^{+}{}_{i-1,0}(y).$$

Hence, $S_{i,j}(x_{i-1},y)=\varphi1^{+}{}_{i-1,0}(y),\ y_{j-1}\le y\le y_j$.

By analogy setting $x=x_i$, $y=y_{j-1}$, $y=y_j$ in (1) we derive other equalities.

Let $1\le s'\le\rho-1$, then:

$$\left.\frac{\partial^{s'}S_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_i}=\left.\frac{\partial^{s'}S1_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_i}+\left.\frac{\partial^{s'}S2_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_i}-\left.\frac{\partial^{s'}S12_{ij}(x,y)}{\partial x^{s'}}\right|_{x=x_i}=$$

$$=\sum_{s=0}^{\rho-1}\varphi1^{+}{}_{i-1,s}(y)\cdot\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i-1,s}(x)\right|_{x=x_{i-1}}+\sum_{s=0}^{\rho-1}\varphi1^{-}{}_{i,s}(y)\cdot\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i,s}(x)\right|_{x=x_{i-1}}+$$

$$\sum_{p=0}^{\rho-1}\left.\frac{\partial^{s'}}{\partial x^{s'}}\varphi2^{+}{}_{j-1,p}(x)\right|_{x=x_{i-1}}\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\left.\frac{\partial^{s'}}{\partial x^{s'}}\varphi2^{-}{}_{j,p}(x)\right|_{x=x_{i-1}}\cdot h2_{j,p}(y)-$$

$$\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,s,p}\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i-1,s}(x)\right|_{x=x_{i-1}}h2_{j-1,p}(y)-$$

$$-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{+-}_{i-1,j,s,p}\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i-1,s}(x)\right|_{x=x_{i-1}}h2_{j,p}(y)-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{-+}_{i,j-1,s,p}\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i,s}(x)\right|_{x=x_{i-1}}h2_{j-1,p}(y)+$$

$$+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{--}_{i,j,s,p}\left.\frac{\partial^{s'}}{\partial x^{s'}}h1_{i,s}(x)\right|_{x=x_{i-1}}h2_{j,p}(y)=\sum_{s=0}^{\rho-1}\varphi1^{+}{}_{i-1,s}(y)\cdot\delta_{i-1,i-1}\delta_{s',s}+\sum_{s=0}^{\rho-1}\varphi1^{-}{}_{i,s}(y)\cdot\delta_{i,i-1}\delta_{s',s}+$$

$$+\sum_{p=0}^{\rho-1}\varphi2^{+(s')}{}_{j-1,p}(x_{i-1})\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}\varphi2^{-(s')}{}_{j,p}(x_{i-1})\cdot h2_{j,p}(y)-$$

$$-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,s,p}\delta_{i-1,i-1}\delta_{s',s}h2_{j-1,p}(y)-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{+-}_{i-1,j,s,p}\delta_{i-1,i-1}\delta_{s',s}h2_{j,p}(y)-$$

$$-\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{-+}_{i,j-1,s,p}\delta_{i,i-1}\delta_{s',s}h2_{j-1,p}(y)+\sum_{s=0}^{\rho-1}\sum_{p=0}^{\rho-1}C^{--}_{i,j,s,p}\delta_{i,i-1}\delta_{s',s}h2_{j,p}(y)=\left|C^{++}_{i-1,j-1,s',p}=\varphi2^{+(s')}{}_{j-1,p}(x_{i-1})\right|=$$

$$=\varphi1^{+}{}_{i-1,s'}(y)+\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,s',p}\cdot h2_{j-1,p}(y)+\sum_{p=0}^{\rho-1}C^{-+}_{i-1,j,s',p}\cdot h2_{j,p}(y)-$$

$$-\sum_{p=0}^{\rho-1}C^{++}_{i-1,j-1,s',p}h2_{j-1,p}(y)-\sum_{p=0}^{\rho-1}C^{+-}_{i-1,j,s',p}h2_{j,p}(y)=\varphi1^{+}{}_{i-1,s'}(y)\,.$$

By analogy properties (2) for $x = x_i$ and (3) are proved.

The proof of Theorem 1 is now complete.

Theorem 2. *Let*

$\varphi1^{-}_{i,s}(y)=\varphi1^{+}_{i,s}(y)=\varphi1_{i,s}(y)$, $s=\overline{0,\mu}, 0\le\mu\le\rho-1$, $\varphi2^{-}_{j,p}(x)=\varphi2^{+}_{j,p}(x)=\varphi2_{j,p}(x)$, $p=\overline{0,\nu}, 0\le\nu\le\rho-1$.

Then the function $S(x,y)=S_{ij}(x,y),\ (x,y)\in\Pi_{ij}$ *has the properties*: $S(x,y)\in C^{\mu,\nu}(D)$,

$$\left.\frac{\partial^{s'}S(x,y)}{\partial x^{s'}}\right|_{x=x_i}=\varphi1_{i,s'}(y),\quad i=\overline{1,m},\ s'=\overline{0,\mu},\ y_{j-1}\le y\le y_j\,, \tag{4}$$

$$\left.\frac{\partial^{p'}S(x,y)}{\partial y^{p'}}\right|_{y=y_j}=\varphi2_{j,p'}(x),\quad j=\overline{1,n},\ p'=\overline{0,\nu},\ x_{i-1}\le x\le x_i\,. \tag{5}$$

Proof. The statement of the theorem follows from the fact that in case $\varphi1_{i,s}(y)\in C^{\rho-1}[x_{i-1},x_i]$, $\varphi2_{j,p}(x)\in C^{\rho-1}[y_{j-1},y_j]$ the function $S_{ij}(x,y)$ belongs to the class $C^{\rho-1,\rho-1}(\Pi_{ij})$ for each element Π_{ij}. Hence, the function $S(x,y)$ is of the class $C^{\rho-1,\rho-1}(\Pi_{ij})$ as well and its derivatives of the order μ,ν respectively stay continuous on the border between the adjacent Π_{ij}, the proof of properties (4), (5) following the same lines as in Theorem 1.

Thus Theorem 2 is proved.

Remark 1. The above implies that in the conditions of Theorem 2 the function $S(x,y)$ has discontinuous partial derivatives of the order greater than μ in x and greater than ν in y.

Remark 2. In general, it is possible that the function $S(x,y)$ and its derivative up to respective orders have discontinuities on the boundaries of one or several elements only.

Theorem 3. *If the functions* $\varphi1^{+}{}_{i,s}(y),\ \varphi1^{-}{}_{i,s}(y)$ *are polynomials (different in general) of degree* $Q\ge2\rho-1$, *and the functions* $\varphi2^{+}{}_{j,p}(x),\ \varphi2^{-}{}_{j,p}(x)$ *are polynomials (different in general) of degree* $Q\ge2\rho-1$, *then the function* $S(x,y)$ *is a piecewise polynomial discontinuous spline, which coincides with a polynomial depending on two variables in each of the rectangles* $\Pi_{ij}\subset D$. *In particular, if* $Q=2\rho-1$, *then* $S(x,y)$ *is a discontinuous piecewise polynomial spline of degree* $2\rho-1$ *in each of the variables* (x,y).

The Proof of the theorem follows from the fact that the Hermit polynomial basic functions are used for constructing the function $S_{ij}(x,y)$. Thus under the assumptions of Theorem 3 $S_{ij}(x,y)$ is polynomials itself. If $Q=2\rho-1$, then $S_{ij}(x,y)$ is a polynomial of degree $2\rho-1$ in each of its variables. If, moreover, the conditions of Theorem 2 do not hold, then the function $S_{ij}(x,y)$ has discontinuities on the borders between different elements Π_{ij}.

Theorem 3 is now proved.

Remark 3. We stress one more time that it is not necessary for $S(x,y)$ to have discontinuities on all the boundaries between all the elements. Moreover, it is not required that the spline has discontinuous derivatives of order $\mu+1,\ \mu+2,\ldots,\rho-1$ and $\nu+1,\nu+2,\ldots,\rho-1$ in x and y respectively on all four sides of each element.

Theorem 4. *Assume that the approximated function* $f(x,y)\in C^{\rho-1,\rho-1}(D\setminus\overline{\Pi_{kl}})$ *and* $\varphi1^{+}{}_{i-1,s}(y)\ne\varphi1^{-}{}_{i,s}(y)$, $\varphi2^{+}{}_{j-1,p}(x)\ne\varphi2^{-}{}_{j,p}(x),\ \ s,p=\overline{0,\rho-1}$. *Then setting*

$$\varphi1^{-}_{i',s}(y)=\varphi1^{+}_{i',s}(y)=f^{(s',0)}(x_{i'},y),\ i'\in\{0,1,\ldots,m\},\ i'\ne i-1,\ i'\ne i,\ 0\le y\le1;$$

$$\varphi2^{-}_{j',p}(x)=\varphi2^{+}_{j',p}(x)=f^{(0,p)}(x,y_{j'}),\ j'\in\{0,1,\ldots,n\},\ j'\ne j-1,\ j'\ne j,\ 0\le x\le1;$$

$$\varphi1^{-}{}_{i-1,s}(y)=\varphi1^{+}{}_{i-1,s}(y)=f^{(s,0)}(x_{i-1},y)\,,\ 0\le y\le y_{j-1}\ \text{or}\ y_j\le y\le1;$$

$$\varphi 2^{-}_{j-1,p}(x)=\varphi 2^{+}_{j-1,p}(x)=f^{(0,p)}(x,y_{j-1})\,,\ 0\le x\le x_{i-1}\ \text{or}\ x_i\le x\le 1\,;$$

$$\varphi 1^{+}_{i-1,s}(y)=f^{(s,0)}(x_{i-1}+0,y),\ \varphi 1^{-}_{i,s}(y)=f^{(s,0)}(x_i-0,y)\,;$$

$$\varphi 1^{-}_{i-1,s}(y)=f^{(s,0)}(x_{i-1}-0,y),\ \varphi 1^{+}_{i,s}(y)=f^{(s,0)}(x_i+0,y)\,;$$

$$\varphi 2^{+}_{j-1,p}(x)=f^{(0,p)}(x,y_{j-1}+0),\ \varphi 2^{-}_{j,p}(x)=f^{(0,p)}(x,y_j-0)\,;$$

$$\varphi 2^{-}_{j-1,p}(x)=f^{(0,p)}(x,y_{j-1}-0),\ \varphi 2^{+}_{j,p}(x)=f^{(0,p)}(x,y_j+0)\,,$$

the function $S(x,y)$ *belongs to the class* $C^{\rho-1,\rho-1}(D)$ *. The function* $S(x,y)$ *and its derivatives up to the order* $\rho-1$ *in each of the variables are discontinuous on the boundaries of the element* Π_{ij} *only.*

Proof. The theorem follows from the fact that the function $S(x,y)$ has continuous derivatives up to the order $\rho-1$ on the boundaries between all the elements other than Π_{ij}, where the discontinuities are possible. Which means that this function belongs to the required class: $S(x,y)\in C^{\rho-1,\rho-1}(D\setminus\overline{\Pi_{kl}})$.

End of the proof of Theorem 4.

Theorem 5. *Let the assumptions of Theorem 4 hold. Then the error of approximation of a discontinuous function* $f(x,y)\in C^{\rho-1,\rho-1}(\Pi_{i,j})$ *by an appropriate discontinuous interlineation spline* $S(x,y)$ *is estimated by the formula:*

$$\left|f(x,y)-S(x,y)\right|=O(\Delta 1^{2\rho}\Delta 2^{2\rho}),\ (x,y)\in\Pi_{kl}\ne\Pi_{i,j}\,,\ \Delta 1=\max_k(x_k-x_{k-1}),\ \Delta 2=\max_l(y_l-y_{l-1})\,;$$

$$\left|f(x,y)-S(x,y)\right|=O(\Delta i^{2\rho}\Delta j^{2\rho}),\ (x,y)\in\Pi_{i,j},\Delta i=x_i-x_{i-1},\ \Delta j=y_j-y_{j-1},\ (i,j)\ne(k,l)\ .$$

Proof. By Definition 1 the operator $S_{ij}(x,y)=S_{ij}f(x,y)$ can be written in the form:

$$S_{ij}f(x,y)=S1_{ij}f(x,y)+S2_{ij}f(x,y)-S12_{ij}f(x,y)\,.$$

By Theorem 3.2.1 in [3] the residue of approximation of the function $f(x,y)$ by the interlineation formulae is given by the operator product of the residue of approximation of $f(x,y)$ by the operators $S1_{ij}f(x,y)$ and $S2_{ij}f(x,y)$:

$$RS_{ij}f(x,y)=(f(x,y)-S_{ij}f(x,y))=(f(x,y)-S1_{ij}f(x,y)-S2_{ij}f(x,y)+S12_{ij}f(x,y))=$$

$$=(f(x,y)-S1_{ij}f(x,y))(f(x,y)-S2_{ij}f(x,y))=RS1_{ij}f(x,y)RS2_{ij}f(x,y)\,.$$

In this case the estimate for the error follows from Corollary 3 of Theorem 3.2.2 in [3].

Theorem 5 is proved.

Example. Consider the rectangular domain $D=\{(x,y):0<x<1,0<y<1\}$. Let $\rho=1,\ m=2,\ n=2$. Introduce the nodes: $x_0=0,\ x_1=0.5,\ \ x_2=1,\ \ y_0=0,\ y_1=0.5,y_2=1$.

Then the partition of the domain D consists of four rectangular elements (fig. 1) given by the formulae:

$$\Pi_{11}=\{(x,y):x_0<x<x_1,y_0<y<y_1\}\,,\ \Pi_{12}=\{(x,y):x_0<x<x_1,y_1<y<y_2\}\,,$$

$$\Pi_{21}=\{(x,y):x_1<x<x_2,y_0<y<y_1\}\,,\ \Pi_{11}=\{(x,y):x_1<x<x_2,y_1<y<y_2\}\,.$$

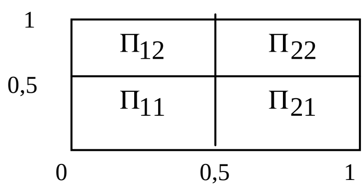

Fig. 1 – Domain D of the approximated function $f(x,y)$.

Let D be the domain of a function $f(x,y)$ we need to approximate. Determine the function $f(x,y)$ at the vertices of the rectangle Π_{ij} as follows:

$\Pi_{11}:\ f^{+,+}(0;0)=f(0+0;0+0)=1,$

$f^{+,-}(0;0.5)=f(0+0;0.5-0)=2,$

$f^{-,-}(0.5;0.5)=f(0.5-0;0.5-0)=1,$

$f^{-,+}(0.5;0)=f(0.5-0;0+0)=2;$

$\Pi_{12}:\ f^{+,+}(0;0.5)=f(0+0;0.5+0)=1,$

$f^{+,-}(0;1)=f(0+0;1-0)=2,$

$f^{-,-}(0.5;1)=f(0.5-0;1-0)=1,$

$f^{-,+}(0.5;0.5)=f(0.5-0;0.5+0)=2;$

$\Pi_{22}: f^{+,+}(0.5;0.5) = f(0.5+0;0.5+0) = 3,$ $\quad$ $\Pi_{21}: f^{+,+}(0.5;0) = f(0.5+0;0+0) = 3,$

$f^{+,-}(0.5;1) = f(0.5+0;1-0) = 4,$ $\quad$ $f^{+,-}(0.5;0.5) = f(0.5+0;0.5-0) = 4,$

$f^{-,-}(1;1) = f(1-0;1-0) = 3,$ $\quad$ $f^{-,-}(1;0.5) = f(1-0;0.5-0) = 3,$

$f^{-,+}(1;0.5) = f(1-0;0.5+0) = 4;$ $\quad$ $f^{-,+}(1;0) = f(1-0;0+0) = 4.$

Construct the discontinuous spline:

$$S(x,y) =$$

$$= \begin{cases} f^{+,+}(0;0)\dfrac{x-x_1}{x_0-x_1}\dfrac{y-y_1}{y_0-y_1} + f^{-,+}(0.5;0)\dfrac{x-x_0}{x_1-x_0}\dfrac{y-y_1}{y_0-y_1} + \\ \quad + f^{+,-}(0;0.5)\dfrac{x-x_1}{x_0-x_1}\dfrac{y-y_0}{y_1-y_0} + f^{-,-}(0.5;0.5)\dfrac{x-x_0}{x_1-x_0}\dfrac{y-y_0}{y_1-y_0}, & (x,y)\in\Pi_{11} \\ f^{+,+}(0;0.5)\dfrac{x-x_1}{x_0-x_1}\dfrac{y-y_2}{y_1-y_2} + f^{-,+}(0.5;0.5)\dfrac{x-x_2}{x_1-x_2}\dfrac{y-y_2}{y_1-y_2} + \\ \quad + f^{+,-}(0;1)\dfrac{x-x_1}{x_0-x_1}\dfrac{y-y_1}{y_2-y_1} + f^{-,-}(0.5;1)\dfrac{x-x_0}{x_1-x_0}\dfrac{y-y_1}{y_2-y_1}, & (x,y)\in\Pi_{12} \\ f^{+,+}(0.5;0)\dfrac{x-x_2}{x_1-x_2}\dfrac{y-y_1}{y_0-y_1} + f^{-,+}(1;0)\dfrac{x-x_1}{x_2-x_1}\dfrac{y-y_1}{y_0-y_1} + \\ \quad + f^{+,-}(0.5;0.5)\dfrac{x-x_2}{x_1-x_2}\dfrac{y-y_0}{y_1-y_0} + f^{-,-}(1;0.5)\dfrac{x-x_1}{x_2-x_1}\dfrac{y-y_0}{y_1-y_0}, & (x,y)\in\Pi_{21} \\ f^{+,+}(0.5;0.5)\dfrac{x-x_2}{x_1-x_2}\dfrac{y-y_2}{y_1-y_2} + f^{-,+}(1;0.5)\dfrac{x-x_1}{x_2-x_1}\dfrac{y-y_2}{y_1-y_2} + \\ \quad + f^{+,-}(0.5;1)\dfrac{x-x_2}{x_1-x_2}\dfrac{y-y_1}{y_2-y_1} + f^{-,-}(1;1)\dfrac{x-x_1}{x_2-x_1}\dfrac{y-y_1}{y_2-y_1}, & (x,y)\in\Pi_{22} \end{cases} =$$

$$= \begin{cases} 1\cdot\dfrac{x-0.5}{-0.5}\dfrac{y-0.5}{-0.5} + 2\cdot\dfrac{x}{0.5}\dfrac{y-0.5}{-0.5} + 2\cdot\dfrac{x-0.5}{-0.5}\dfrac{y}{0.5} + 1\cdot\dfrac{x}{0.5}\dfrac{y}{0.5}, & (x,y)\in\Pi_{11} \\ 1\cdot\dfrac{x-0.5}{-0.5}\dfrac{y-1}{0.5-1} + 2\cdot\dfrac{x-1}{0.5-1}\dfrac{y-1}{0.5-1} + 2\cdot\dfrac{x-0.5}{-0.5}\dfrac{y-0.5}{1-0.5} + 1\cdot\dfrac{x}{0.5}\dfrac{y-0.5}{1-0.5}, & (x,y)\in\Pi_{12} \\ 3\cdot\dfrac{x-1}{0.5-1}\dfrac{y-0.5}{-0.5} + 4\cdot\dfrac{x-0.5}{1-0.5}\dfrac{y-0.5}{-0.5} + 4\cdot\dfrac{x-1}{0.5-1}\dfrac{y}{0.5} + 3\cdot\dfrac{x-0.5}{1-0.5}\dfrac{y}{0.5}, & (x,y)\in\Pi_{21} \\ 3\cdot\dfrac{x-1}{0.5-1}\dfrac{y-1}{0.5-1} + 4\cdot\dfrac{x-0.5}{1-0.5}\dfrac{y-1}{0.5-1} + 4\cdot\dfrac{x-1}{0.5-1}\dfrac{y-0.5}{1-0.5} + 3\cdot\dfrac{x-0.5}{1-0.5}\dfrac{y-0.5}{1-0.5}, & (x,y)\in\Pi_{22} \end{cases} =$$

$$= \begin{cases} 2x+2y-8xy+1, & (x,y)\in\Pi_{11} \\ -10x-6y+8+8xy, & (x,y)\in\Pi_{12} \\ -4xy+2x+2y+2, & (x,y)\in\Pi_{21} \\ -8xy+6x+6y-1, & (x,y)\in\Pi_{22} \end{cases}.$$

The function $S(x,y)$ has the following trace on the boundary between the elements Π_{11} and Π_{21} for $x < x_1$:

$$S(x,y) = S(x_1-0,y) = S_{11}(x_1,y) = f^{-,+}(0.5;0)\frac{y-y_1}{y_0-y_1} + f^{-,-}(0.5;0.5)\frac{y-y_0}{y_1-y_0}, \quad y_0 \le y \le y_1.$$

By analogy:

$$S(x,y) = S(x_1+0,y) = S_{21}(x_1,y) = f^{+,+}(0.5;0)\frac{y-y_1}{y_0-y_1} + f^{+,-}(0.5;0.5)\frac{y-y_0}{y_1-y_0}, \quad y_0 \le y \le y_1.$$

Hence, if $f^{-,+}(0.5,0) \ne f^{+,+}(0.5,0)$, then the spline has a discontinuity at the point $(0.5;0)$. Moreover, if $f^{+,+}(0.5;0.5) \ne f^{+,-}(0.5;0.5)$, then the spline is discontinuous on the line $x = 0.5$, $y_0 \le y \le y_1$.

Let the approximated function be given by the formula:

$$f(x,y) = S_{ij}(x,y) + \frac{(x-x_{i-1})(x_i-x)(y-y_{j-1})(y_j-y)}{4}, (x,y)\in\Pi_{i,j}, i,j=1,2.$$

Then it has the partial derivative $f^{2,2}(x,y) \equiv 1$, $\forall (x,y)\in\Pi_{ij}$ in each of the four rectangular elements Π_{ij} of its domain. Hence, the error of approximation of such a discontinuous function by the discontinuous spline constructed above satisfies the inequality:

$$\max_{(x,y)\in\Pi_{ij}}\left|f(x,y)-S_{i,j}(x,y)\right|\leq f^{(2,2)}(\xi,\eta)\cdot\frac{\Delta i^2\Delta j^2}{2!\cdot 2!}=1\cdot\frac{\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2}{2!\cdot 2!}=\frac{1}{64}\approx 0.016\,.$$

Conclusions. The paper presents a general approach to constructing discontinuous interlineation splines, which particular cases are discontinuous splines and splines continuously differentiable up to some order in each variable. The theorems on interlineation and approximation properties of such discontinuous structures are formulated and proved. In particular, the mentioned properties lead the authors to the conclusion that the function of two variables discontinuous at some points or on some lines should be approximated by discontinuous interlineation splines. Such an approach enables one to get the approximation errors of the same order of accuracy in each element of the partition, which is peculiar for continuously differentiable interlineation splines.

As the next step of their research, the authors plan to develop the theory of discontinuous interlineation splines on the domains of complicated shape bounded by the arcs of known curves.

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MATHEMATICAL MODELS AND INFORMATIONAL TECHNOLOGIES OF INNOVATIVE PROJECT ARRANGEMENT IN THE STAKEHOLDERS' SYSTEM

During the involvement of innovative projects into knowledge-intensive high-tech enterprises, the process of creating a system of interested stakeholder management becomes vital. The given work contains the results of conducted analysis concerning the problem of innovative potential management of high-tech enterprises. The necessity of the analysis of informational technologies in the conditions of the non-equilibrium economy is considered. Various models of project management in the system of stakeholders are presented in the work. The stages of Nicholas model are considered. A mathematical model is proposed for the management and investors of the project, in terms of maximizing profits under specified constraints.

Key words: innovative project, stakeholders, Mitchell's model, ASC model, informational technologies.

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МАТЕМАТИЧНІ МОДЕЛІ І ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ ОРГАНІЗАЦІЇ ІННОВАЦІЙНИХ ПРОЕКТІВ У СИСТЕМІ СТЕЙКХОЛДЕРІВ

При залученні інноваційного проекту в наукоміскі високотехнологічні підприємства актуальним процесом стає створення системи управління зацікавленими учасниками. Проведено аналіз проблеми управління інноваційним потенціалом наукоміських підприємств. Розглянуто необхідність аналізу інформаційних технологій в умовах нерівноважної економіки. Розглянуто різні моделі управління проектами в системі зацікавлених осіб. Розглянуто етапи використання моделі Ніколаса. Запропоновано математичну модель для керівництва та інвесторів проекту, в умовах максимізації прибутку при заданих обмеженнях.

Ключові слова: інноваційний проект, стейкхолдери, модель Мітчелла, ASC модель, інформаційні технології.

Р. В. ПЕТРОВА, О. И. ЛЮБИЧЕВА, А. И. МОРОЗОВА

МАТЕМАТИЧЕСКИЕ МОДЕЛИ И ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ ОРГАНИЗАЦИИ ИННОВАЦИОННЫХ ПРОЕКТОВ В СИСТЕМЕ СТЕЙКХОЛДЕРОВ

При вовлечении инновационного проекта в наукоемкие высокотехнологичные предприятия актуальным процессом становится создание системы управления заинтересованными участниками. Проведен анализ проблемы управления инновационным потенциалом наукоемких предприятий. Рассмотрена необходимость анализа информационных технологий в условиях неравновесной экономики. Рассмотрены различные модели управления проектами в системе заинтересованных лиц. Рассмотрены этапы использования модели Николаса. Предложена математическая модель для руководства и инвесторов проекта, в условиях максимизации прибыли при заданных ограничениях.

Ключевые слова: инновационный проект, стейкхолдеры, модель Митчелла, ASC модель, информационные технологии.

Introduction. The analysis of practical innovative development has shown that the management of innovative potential requires an appropriate update not only in the field of creating a technological platform and innovative products but also in terms of creating an organizational and economic mechanism for managing innovative activity. Of particular importance is the creation of a stakeholder management system when high-tech enterprises and organizations are involved in an innovative project. In order to organize effective project activities with the participation of all stakeholders, it is necessary to carry out a set of interrelated mathematical models.

Problem statement. The methodological problem of managing the innovative potential of knowledge-intensive companies is not only the establishment of stakeholder groups, but also the analysis of informational technology and the development of a network model of various resource exchange, the assessment of network density and centrality in a non-equilibrium economy.